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## Sharp-Crested Weirs for Open Channel Flow Measurement

Course No: C02-022
Credit: 2 PDH

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# Sharp-Crested Weirs for Open Channel Flow Mesurement 

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## COURSE CONTENT

## 1. Introduction

A weir is basically an obstruction in an open channel flow path. Weirs are commonly used for measurement of open channel flow rate. A weir functions by causing water to rise above the obstruction in order to flow over it. The height of water above the obstruction correlates with the flow rate, so that measurement of the height of the flowing water above the top of the weir can be used to determine the flow rate by the use of an equation, graph or table. The top of the weir, which is used as the reference level for the height of water flowing over it, is called the crest of the weir. Weirs are typically classified as being either sharp-crested or broad-crested. This course is devoted to the more widely used sharp-crested weir. The major emphasis is on the calculations used for flow rate over various types of sharp-crested weirs. There is also information about guidelines for installation and use of sharp-crested weirs.


Sharp Crested Weirs

## 2. Topics Covered in This Course

I. Sharp-Crested Weir Background
II. V-notch Weirs
III. Suppressed Rectangular Weirs
IV. Contracted Rectangular Weirs
V. Cipolletti Weirs
VI. Installation and Use Guidelines for Sharp-Crested Weirs
VII. Summary
VIII. References and Websites

## 3. Sharp-Crested Weir Background

A sharp-crested weir consists of a vertical flat plate with a sharp edge at the top (the crest), placed in an open channel so that the liquid must flow over the crest in order to drop into the pool below the weir. Figure 1 below shows a longitudinal section representing flow over a sharp-crested weir.


Figure 1. Longitudinal Section, Flow Over a Sharp-crested Weir

Some of the terminology used in connection with sharp-crested weirs, illustrated in Figure 1, is summarized here:

Drawdown, as shown in the figure, occurs upstream of the weir plate due to the acceleration of the water as it approaches the weir.

The term Nappe is used for the sheet of water flowing over the weir.
Free flow is the flow condition over a sharp-crested weir when there is free access of air under the nappe.

The Velocity of approach is equal to the discharge, Q, divided by the crosssectional area of flow at the head measuring station, which should be upstream far enough that it is not affected by the drawdown.

Submerged flow or a submerged weir occurs when downstream water rises above the weir crest elevation.

The equations to be discussed for sharp-crested weirs all require free flow conditions. Accurate measurement of flow rate is not possible under submerged flow conditions.

Four common sharp-crested weir shapes will be covered in some detail in this course. They are the four shapes shown in Figure 2 on the next page: V-notch, suppressed rectangular, contracted rectangular, and cipolletti.

The source for the equations, graphs, etc used for calculating flow rate over a weir based on measured values such as the head over the weir and various weir parameters, is the 2001 revision, of the 1997 third edition, of the Water Measurement Manual, produced by the U.S. Dept. of the Interior, Bureau of Reclamation. This manual is available for free download and/or free online use at: http://www.usbr.gov/pmts/hydraulics lab/pubs/wmm/index.htm . The Water Measurement Manual has very extensive coverage of water flow rate measurement. It is primarily about open channel flow, but has some coverage of pipe flow (closed conduit pressurized flow) as well.


Figure 2. Common Sharp-crested Weir Shapes

## 4. V-Notch Weirs

The V-notch, sharp-crested weir is especially good for measuring low flow rates. The flow area decreases as H increases, so a reasonable head is developed even at a very small flow rate. A V-notch weir (sometimes called a triangular weir) is shown in Figure 2 (a) and in Figure 3, below.

Equation (1) below is given by the Bureau of Reclamation, in their Water Measurement Manual, as an equation suitable for use with a fully contracted, $\mathbf{9 0}^{\mathbf{}}$ V-notch, sharp-crested weir if it meets the conditions summarized below the equation.
(U.S. units: Q in cfs, H in ft) $\quad \mathbf{Q}=\mathbf{2 . 4 9} \mathbf{H}^{2.48}$

Subject to: $\mathbf{P} \geq \mathbf{2} \mathbf{H}_{\text {max }}, \quad \mathbf{S} \geq \mathbf{2} \mathbf{H}_{\text {max }}, \quad \mathbf{0 . 2} \mathbf{f t} \leq \mathbf{H} \leq \mathbf{1 . 2 5} \mathbf{f t}$

Where: $\quad \mathrm{H}_{\text {max }}$ is the maximum head expected over the weir
P is the height of the V-notch vertex above the channel invert
S is the distance from the channel wall to the to the V -notch edge at the top of the overflow

Figure 3, below, illustrates the parameters S, P, \& H.


Figure 3. Fully Contracted V-notch Weir

The Bureau of Land Reclamation in the their Water Measurement Manual recommends 0.2 ft to 1.25 ft as the useable head over a V-notch weir for accurate measurement of flow rate.

Example \#1: Calculate the minimum and maximum flow rates covered by the recommended range of 0.2 ft to 1.25 ft for the head over a fully contracted $90^{\circ} \mathrm{V}$ notch weir. (Note: in order to be fully contracted, $\mathrm{P} \geq 2 \mathrm{H}_{\max }=2.5 \mathrm{ft}$.)

Solution: Substituting values of H into equation (1) gives:

$$
\begin{aligned}
& \mathrm{Q}_{\min }=(2.49)\left(0.2^{2.48}\right)=\underline{\mathbf{0 . 0 4 6} \mathbf{c f s}}=\mathbf{Q}_{\text {min }} \\
& \mathrm{Q}_{\max }=(2.49)\left(1.25^{2.48}\right)=\underline{\mathbf{4 . 3 3} \mathbf{c f s}}=\mathbf{Q}_{\text {max }}
\end{aligned}
$$

Notch angles other than $9 \mathbf{0 0}^{\circ}$ in a V-notch, sharp-crested weir require the use of the Kindsvater-Carter equation, as given in Water Measurement Manual:

$$
\begin{equation*}
\mathrm{Q}=4.28 \mathrm{Ce} \operatorname{Tan}\left(\frac{\theta}{2}\right)(\mathrm{H}+\mathrm{k})^{5 / 2} \tag{2}
\end{equation*}
$$

Where: $\quad \mathrm{Q}$ is the discharge over the weir in cfs
Ce is the effective discharge coefficient
H is the head over the weir in ft
k is a head correction factor
$\theta$ is the angle of the V -notch

Ce , the effective weir coefficient, and k , the head correction factor, are both functions of only $\theta$ if the $V$-notch weir is fully contracted $\left(\mathbf{P} \geq \mathbf{2} \mathbf{H}_{\text {max }} \boldsymbol{\&}\right.$ $\mathbf{S} \geq \mathbf{2} \mathbf{H}_{\text {max }}$ ). Figure 3 on page 4 illustrates the parameters P and H . Ce can be obtained from figure $4^{*}$ or equation (3)*, and k , can be obtained from figure $5^{*}$ or equation (4)*.
*Figure 4, equation (3), Figure 5, and eqution (4) are from LMNO Engineering, Research and Software, Ltd website, at: http://www.Imnoeng.com/Weirs


Figure 4. Effective V-notch Weir Coefficient, Ce

$$
\begin{equation*}
\mathrm{Ce}=0.607165052-(0.000874466963) \theta+\left(6.10393334 \times 10^{-6}\right) \theta^{2} \tag{3}
\end{equation*}
$$



Figure 5. V-notch Weir, Head Correction Factor, k

$$
\mathrm{k}=0.0144902648-(0.00033955535) \theta
$$

$$
\begin{equation*}
+\left(3.29819003 \times 10^{-6}\right) \theta^{2}-\left(1.06215442 \times 10^{-8}\right) \theta^{3} \tag{4}
\end{equation*}
$$

Example \#2: Use the Kindsvater-Carter equation to estimate the flow rate through a fully contracted V-notch weir for a head of 0.2 feet and for a head of 1.25 feet.
a) with a notch angle of $90^{\circ}$
b) with a notch angle of $60^{\circ}$
c) with a notch angle of $40^{\circ}$

Solution: Part a) From equation (3) \& (4), with $\theta=90^{\circ}$ :

$$
\mathrm{Ce}=0.5779 \& \mathrm{k}=0.0029
$$

Substituting into equation (2), with $\mathrm{H}=0.2 \mathrm{ft}$, gives: $\quad\left(\boldsymbol{\theta}=\mathbf{9 0}^{\mathbf{0}}, \mathbf{H}=\mathbf{0 . 2} \mathbf{f t}\right)$

$$
\mathrm{Q}=(4.28)(0.5779)\left[\tan \left(45^{\circ}\right)\right]\left[(0.2+0.0029)^{5 / 2}\right]=\underline{\mathbf{0 . 0 4 6} \mathbf{c f s}}
$$

Substituting into equation (2), with $\mathrm{H}=1.25 \mathrm{ft}$, gives: $\quad\left(\boldsymbol{\theta}=\mathbf{9 0}^{\boldsymbol{\circ}}, \mathbf{H}=\mathbf{1 . 2 5} \mathbf{f t}\right)$

$$
\mathrm{Q}=(4.28)(0.5779)\left[\tan \left(45^{\circ}\right)\right]\left[(1.25+0.0029)^{5 / 2}\right]=\underline{\mathbf{4 . 3 5} \mathbf{~ c f s}}
$$

Part b) From equation (3) \& (4), with $\theta=60^{\circ}$ :

$$
\mathrm{Ce}=0.5767 \& \mathrm{k}=0.0037
$$

Substituting into equation (2), with $\mathrm{H}=0.2 \mathrm{ft}$, gives: $\quad\left(\boldsymbol{\theta}=\mathbf{6 0}^{\mathbf{o}}, \mathbf{H}=\mathbf{0} . \mathbf{2 f t}\right)$

$$
\mathrm{Q}=(4.28)(0.5767)\left[\tan \left(30^{\circ}\right)\right]\left[(0.2+0.0037)^{5 / 2}\right]=\underline{\mathbf{0 . 0 2 7} \mathbf{c f s}}
$$

Substituting into equation (2), with $\mathrm{H}=1.25 \mathrm{ft}$, gives: $\quad\left(\boldsymbol{\theta}=\mathbf{6 0}^{\circ}, \mathbf{H}=\mathbf{1 . 2 5} \mathbf{f t}\right)$

$$
\mathrm{Q}=(4.28)(0.5767)\left[\tan \left(30^{\circ}\right)\right]\left[(1.25+0.0037)^{5 / 2}\right]=\underline{\mathbf{2} .51} \mathbf{~ c f s}
$$

Part c) From equation (3) \& (4), with $\theta=40^{\circ}: \mathrm{Ce}=0.5820 \& \mathrm{k}=0.0051$

Substituting into equation (2), with $\mathrm{H}=0.2 \mathrm{ft}$, gives: $\quad\left(\boldsymbol{\theta}=\mathbf{4 0}^{\boldsymbol{o}}, \mathbf{H}=\mathbf{0 . 2} \mathbf{f t}\right)$

$$
\mathrm{Q}=(4.28)(0.5820)\left[\tan \left(20^{\circ}\right)\right]\left[(0.2+0.0051)^{5 / 2}\right]=\underline{\mathbf{0 . 0 1 7} \mathbf{c f s}}
$$

Substituting into equation (2), with $\mathrm{H}=1.25 \mathrm{ft}$, gives: $\quad\left(\boldsymbol{\theta}=\mathbf{4 0}^{\boldsymbol{}}, \mathbf{H}=\mathbf{1 . 2 5} \mathbf{f t}\right)$

$$
\mathrm{Q}=(4.28)(0.5820)\left[\tan \left(20^{\circ}\right)\right]\left[(1.25+0.0051)^{5 / 2}\right]=\underline{\mathbf{1 . 6 0} \mathbf{c f s}}
$$

Comment: This example illustrates the use of equations (2), (3), \& (4) for V-notch weirs with notch angles other than $90^{\circ}$. Note that the answers for the $90^{\circ}$ notch angle are nearly the same as those from the simpler Equation (1) used for the same minimum and maximum H values in Example \#1.

## 5. Suppressed Rectangular Weirs

The weir goes across the entire channel width for a suppressed rectangular, sharp-crested weir, as shown in the diagram below. The end contractions, as shown in Figure 2 (c) for a contracted rectangular weir, are suppressed (not present) in this configuration; hence the name suppressed rectangular weir.

suppressed rectangular weir
C.E. Kindsvater and R.W. Carter presented a paper on the discharge characteristics of thin-plate rectangular weirs (see ref. \#3 at the end of this course). The recommendations in that paper have come to be known as The Kindsvater-Carter method, which is recommended in Water Measurement Manual, as a method with flexibility and a wide range of applicability for rectangular sharp-crested weirs. The Kindsvater-Carter Method will be presented and discussed here. A simpler equation, which is suitable under particular conditions, will then be presented as an alternative.

The Kindsvater-Carter equation for a rectangular, sharp-crested weir (suppressed or contracted) is:

$$
\begin{equation*}
\mathrm{Q}=\mathrm{C}_{\mathrm{e}} \mathrm{~L}_{\mathrm{e}} \mathrm{H}_{\mathrm{e}}^{3 / 2} \tag{5}
\end{equation*}
$$

where:
Q is the discharge (flow rate over the weir) $\mathrm{in}^{\mathrm{ft}} / \mathrm{s}$.
e is a subscript denoting "effective".
$\mathrm{C}_{\mathrm{e}}$ is the effective coefficient of discharge in $\mathrm{ft}^{1 / 2} / \mathrm{s}$.
$\mathrm{L}_{\mathrm{e}}=\mathrm{L}+\mathrm{k}_{\mathrm{b}}$
$\mathrm{H}_{\mathrm{e}}=\mathrm{H}+\mathrm{k}_{\mathrm{h}}$
In these relationships:
$\mathrm{k}_{\mathrm{b}}$ is a correction factor to obtain effective weir length.
L is the length of the weir crest in ft .
B is the average width of the approach channel in ft .
H is the head measured above the weir crest in ft .
$\mathrm{k}_{\mathrm{h}}$ is a correction factor having a value of 0.003 ft

The factor $\mathrm{k}_{\mathrm{b}}$ depends on the ratio of crest length to average width of approach channel (L/B). Values of $\mathrm{k}_{\mathrm{b}}$ for ratios of $\mathrm{L} / \mathrm{B}$ from 0 to 1 are given in Figure 6 on the next page (from Water Measurement Manual).


Figure 6. $\mathrm{k}_{\mathrm{b}}$ as a function of $\mathrm{L} / \mathrm{B}$ (as given in Water Measurement Manual)


Figure 7. $\mathrm{C}_{\mathrm{e}}$ as a function of $\mathrm{H} / \mathrm{P} \& \mathrm{~L} / \mathrm{B}$ for Rect. Sharp-crested Weir

As shown in Figure 1, P is the vertical distance to the weir crest from the approach pool invert. (NOTE: The term "invert" means the inside, upper surface of the channel.)

The effective coefficient of discharge, $\mathrm{C}_{\mathrm{e}}$, is a function of relative depth, $\mathrm{H} / \mathrm{P}$ and relative width, $\mathrm{L} / \mathrm{B}$, of the approach channel. Figure 7 on the previous page (from Water Measurement Manual), gives a set of curves for $\mathrm{C}_{\mathrm{e}}$ in terms of H/P and L/P. Also $\mathrm{C}_{\mathrm{e}}$ can be calculated from equation (6) and Table 1.

The straight lines in figure 7 can be represented by an equation of the form:

$$
\begin{equation*}
\mathrm{C}_{\mathrm{e}}=\mathrm{C}_{1}\left(\frac{\mathrm{H}}{\mathrm{P}}\right)+\mathrm{C}_{2} \tag{6}
\end{equation*}
$$

Where the constants, $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$, in equation (6) are functions of $\mathrm{L} / \mathrm{B}$ as shown in Table 1, (from Water Measurement Manual) given below.

Table 1. Values of $\mathrm{C}_{1} \& \mathrm{C}_{2}$ for equation (6) (from Water Measurement Manual)

| $\mathrm{L} / \mathrm{B}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ |
| :---: | :---: | :---: |
| 0.2 | -0.0087 | 3.152 |
| 0.4 | 0.0317 | 3.164 |
| 0.5 | 0.0612 | 3.173 |
| 0.6 | 0.0995 | 3.178 |
| 0.7 | 0.1602 | 3.182 |
| 0.8 | 0.2376 | 3.189 |
| 0.9 | 0.3447 | 3.205 |
| 1.0 | 0.4000 | 3.220 |

The Kindsvater-Carter equation for a rectangular, sharp-crested weir, in general, as presented on the last three pages is rather complicated. It simplifies a great
deal, however, for a suppressed rectangular weir $(\mathrm{L}=\mathrm{B}$ or $\mathrm{L} / \mathrm{B}=1.0$, so $\mathrm{C}_{1}=0.4000$ and $\mathrm{C}_{2}=3.220$ ). Equation (6) thus becomes:

$$
\mathrm{C}_{\mathrm{e}}=0.4000\left(\frac{\mathrm{H}}{\mathrm{P}}\right)+3.220
$$

From Figure $6(L / B=1): k_{b}=-0.003, \quad$ so: $\quad L_{e}=L+k_{b}=L-0.003$

As given above: $\mathrm{k}_{\mathrm{h}}=0.003$, so: $\mathrm{H}_{\mathrm{e}}=\mathrm{H}+\mathrm{k}_{\mathrm{h}}=\mathrm{H}+0.003$

Putting the expressions for $\mathrm{C}_{\mathrm{e}}, \mathrm{L}_{\mathrm{e}}, \& \mathrm{H}_{\mathrm{e}}$ into equation (5), gives the following Kindsvater-Carter equation for a suppressed rectangular, sharp-crested weir:

$$
\begin{equation*}
\mathrm{Q}=\left(0.4000\left(\frac{\mathrm{H}}{\mathrm{P}}\right)+3.220\right)(\mathrm{L}-0.003)(\mathrm{H}+0.003)^{3 / 2} \tag{7}
\end{equation*}
$$

(Note: Equation (7) is a dimensional equation. Q is in cfs and $\mathrm{H}, \mathrm{P}, \& \mathrm{~L}$ are in ft.)

The Bureau of Reclamation, in their Water Measurement Manual, gives equation (8) below, as an equation suitable for use with suppressed rectangular, sharpcrested weirs if the conditions noted below are met:
(U.S. units: Q in cfs, $\mathrm{B} \& \mathrm{H}$ in ft): $\quad \mathbf{Q}=\mathbf{3 . 3 3} \mathbf{B} \mathbf{H}^{3 / 2}$

To be used only if:

$$
\frac{\mathrm{H}}{\mathrm{P}} \leq 0.33 \quad \& \quad \frac{\mathrm{H}}{\mathrm{~B}} \leq 0.33
$$

Equation (8) can be converted to S.I. units to give the following:
(S.I. units: Q in $\mathrm{m}^{3} / \mathrm{s}, \mathrm{B} \& \mathrm{H}$ in m ): $\quad \mathbf{Q}=\mathbf{1 . 8 4} \mathbf{B ~ H}^{3 / 2}$

To be used only if:

$$
\frac{\mathrm{H}}{\mathrm{P}} \leq 0.33 \quad \& \quad \frac{\mathrm{H}}{\mathrm{~B}} \leq 0.33
$$

Example \#3: There is a suppressed rectangular weir in a 3 ft wide rectangular channel. The weir crest is 1.5 ft above the channel invert. Calculate the flow rate for $\mathrm{H}=0.2,0.3,0.5,0.7,1.0 \mathrm{ft}, \& 1.3 \mathrm{ft}$, using equations (7) \& (8).

Solution: From the problem statement, $\mathrm{L}=\mathrm{B}=3 \mathrm{ft}$, and $\mathrm{P}=1.5 \mathrm{ft}$. The solution consists of substituting values of $\mathrm{B}, \mathrm{H}, \mathrm{L}$, and $(\mathrm{H} / \mathrm{P})$ into equation (7) and equation (8) and calculating Q for each value of H . The calculations were made with Excel. The results from the spreadsheet are shown in the table below, where $\mathrm{Q}_{7}$ is the flow rate calculated from equation (7) and $\mathrm{Q}_{8}$ is the flow rate calculated from equation (8).

Summary of Solution to Example \#3

| $\underline{\mathrm{H}, \mathrm{ft}}$ | $\underline{0.2}$ | $\underline{0.3}$ | $\underline{0.5}$ | $\underline{0.7}$ | $\underline{1}$ | $\underline{1.5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{H} / \mathrm{B}$ | 0.067 | 0.100 | 0.167 | 0.233 | 0.333 | 0.500 |
| $\mathrm{H} / \mathrm{P}$ | 0.13 | 0.20 | 0.33 | 0.47 | 0.67 | 1.00 |
| Q $_{7}, \mathbf{c f s}$ | $\mathbf{0 . 8 9 7}$ | $\mathbf{1 . 6 5 0}$ | $\mathbf{3 . 5 8 5}$ | $\mathbf{6 . 0 1 8}$ | $\mathbf{1 0 . 5 0}$ | $\mathbf{1 9 . 9 9}$ |
| $\mathbf{Q}_{8}$, cfs | $\mathbf{0 . 8 9 4}$ | $\mathbf{1 . 6 4 2}$ | $\mathbf{3 . 5 3 2}$ | $\mathbf{5 . 8 5 1}$ | $\mathbf{9 . 9 9 0}$ | $\mathbf{1 8 . 3 5}$ |
| \% diff | $0.4 \%$ | $0.5 \%$ | $1.5 \%$ | $2.8 \%$ | $4.8 \%$ | $8.2 \%$ |

Discussion of results: The summary table shows values for $\mathrm{H} / \mathrm{B}$ and $\mathrm{H} / \mathrm{P}$ for each value of H . This facilitates comparison of the results from the two equations while taking into account whether or not the conditions specified for the use of
equation ( 8 ) were met $(\mathrm{H} / \mathrm{B} \leq 0.33$ and $\mathrm{H} / \mathrm{P} \leq 0.33)$. The results show good agreement between the flow rates calculated from the simpler equation (8) and the more comprehensive equation (7) for the three smaller values of H (for which $\mathrm{H} / \mathrm{B} \leq 0.33$ and $\mathrm{H} / \mathrm{P} \leq 0.33$ ). For the larger values of H , the disagreement between the two equations becomes greater as the values of H become larger.

Conclusions: For a suppressed rectangular, sharp-crested weir, equation (8), $\mathbf{Q}=\mathbf{3 . 3 3 B H} \mathbf{B}^{3 / 2}$, may be used if $\mathrm{H} / \mathrm{P} \leq 0.33 \& \mathrm{H} / \mathrm{B} \leq 0.33$. For $\mathrm{H} / \mathrm{P}>0.33$ or $\mathrm{H} / \mathrm{B}>0.33$, the more comprehensive Kindsvater-Carter equation [equation (7)] should be used.

Example \#4: Calculate the minimum and maximum flow rates covered by the range of 0.2 ft to 1.25 ft for the head over a suppressed rectangular weir in a 2 ft wide rectangular channel. Assume that $\mathrm{P}=2.5 \mathrm{ft}$, as with the V -notch weir in Example \#1, so that the suppressed rectangular weir is fully contracted from the channel bottom.

Solution: Equation (4) can be used for $\mathrm{H}=0.2 \mathrm{ft}$, because $\mathrm{H} / \mathrm{P}<0.33$ and $\mathrm{H} / \mathrm{B}<$ 0.33 , however for $\mathrm{H}=1.25 \mathrm{ft}, \mathrm{H} / \mathrm{B} \& \mathrm{H} / \mathrm{P}$ are both greater than 0.33 , so equation (3) must be used. The calculations are shown below:

$$
\mathrm{Q}_{\text {min }}=(3.33)(2)\left(0.2^{3 / 2}\right)=\underline{\mathbf{0 . 5 9 6} \mathbf{c f s}}=\mathbf{Q}_{\text {min }}
$$

$\mathrm{Q}_{\text {max }}=[3.32+0.40(1.25 / 2.5)](2-0.003)\left[(2+0.003)^{3 / 2}\right]=\underline{\mathbf{1 0 . 4 2} \mathbf{c f s}}=\mathbf{Q}_{\text {max }}$
Comments: As shown by Example \#1 and Example \#4, the 2 ft rectangular weir can carry more than twice the flow rate that the V-notch weir can carry for the same head above the channel bottom, however the V-notch weir can measure a much smaller flow rate than is possible with the rectangular weir.

## 6. Contracted Rectangular Weirs

A contracted rectangular sharp-crested weir, as shown in Figure 2c and in the figure on the next page, has weir length less than the width of the channel. This type of rectangular weir is sometimes called an unsuppressed rectangular weir.

contracted rectangular weir

For a contracted rectangular, sharp-crested weir (L/B $<1$ ), the KindsvaterCarter equation becomes:

$$
\begin{equation*}
\mathrm{Q}=\left(\mathrm{C}_{1}\left(\frac{\mathrm{H}}{\mathbf{P}}\right)+\mathrm{C}_{2}\right)(\mathbf{L}-0.003)\left(\mathbf{H}+\mathbf{k}_{\mathrm{b}}\right)^{3 / 2} \tag{9}
\end{equation*}
$$

Where: $\quad C_{1}$ and $C_{2}$ can be obtained from Table 1 and $k_{b}$ can be obtained from Figure 6, for a known value of L/B. (Table 1 is on page 12 and Figure 6 is on page 10.)

Equation (10) below is a commonly used, simpler equation for a fully contracted rectangular weir. The Bureau of Reclamation, in its Water Measurement Manual, specifies this equation as being suitable for use if the conditions noted below the equation are met.
(U.S. units: Q in $\mathrm{cfs}, \mathrm{L} \& \mathrm{H}$ in ft ): $\quad \mathbf{Q}=\mathbf{3 . 3 3}(\mathbf{L}-\mathbf{0 . 2} \mathbf{H})\left(\mathbf{H}^{3 / 2}\right)$

For use of equation (10), the following conditions must be met:
i) Weir must be fully contracted, i.e.: $\quad \mathbf{B}-\mathbf{L} \geq \mathbf{4} \mathbf{H}_{\text {max }}$ and $\quad \mathbf{P} \geq \mathbf{2} \mathbf{H}_{\text {max }}$
ii) $\quad \mathbf{H} / \mathrm{L} \leq \mathbf{0 . 3 3}$

The equivalent equation for S.I. units is:
(S.I. units: Q in $\mathrm{m}^{3} / \mathrm{s}, \mathrm{L} \& \mathrm{H}$ in m$): \quad \mathbf{Q}=\mathbf{1 . 8 4}(\mathbf{L}-\mathbf{0 . 2} \mathbf{H})\left(\mathbf{H}^{3 / 2}\right)$

The two conditions given for equation (10) apply to equation (11) also.

Example \#5: Consider a contracted rectangular weir in a rectangular channel with $B, L, H, \& P$ having each of the following sets of values.
a) Determine whether the conditions for use of equation (10) are met for each set of values.
b) Calculate the flow rate, Q , using equations (9) \& (10) for each set of values.
i) $\quad \mathrm{B}=4 \mathrm{ft}, \quad \mathrm{L}=2 \mathrm{ft}, \quad \mathrm{H}=0.5 \mathrm{ft}, \quad \mathrm{P}=1 \mathrm{ft}$
ii) $\quad \mathrm{B}=6 \mathrm{ft}, \quad \mathrm{L}=2.4 \mathrm{ft}, \quad \mathrm{H}=0.5 \mathrm{ft}, \quad \mathrm{P}=1.2 \mathrm{ft}$
iii) $\mathrm{B}=6 \mathrm{ft}, \quad \mathrm{L}=3 \mathrm{ft}, \quad \mathrm{H}=0.75 \mathrm{ft}, \quad \mathrm{P}=2.4 \mathrm{ft}$
iv) $\mathrm{B}=6 \mathrm{ft}, \quad \mathrm{L}=4.2 \mathrm{ft}, \quad \mathrm{H}=1.2 \mathrm{ft}, \quad \mathrm{P}=2 \mathrm{ft}$
v) $\mathrm{B}=6 \mathrm{ft}, \quad \mathrm{L}=3 \mathrm{ft}, \quad \mathrm{H}=2 \mathrm{ft}, \quad \mathrm{P}=2.4 \mathrm{ft}$

Solution: a) The required conditions for use of equation (10) are:

1) $\mathrm{B}-\mathrm{L} \geq 4 \mathrm{H}_{\max }$,
2) $\mathrm{P} \geq 2 \mathrm{H}_{\max } \quad \&$
3) $\mathrm{H} / \mathrm{L} \leq 0.33$
i) $\quad \mathrm{B}-\mathrm{L}=2 \mathrm{ft} \& 4 \mathrm{H}=2 \mathrm{ft}$, so condition 1 is barely met
$\mathrm{P}=1 \mathrm{ft} \& 2 \mathrm{H}=1 \mathrm{ft}$, so condition 2 is barely met
$\mathrm{H} / \mathrm{L}=0.5 / 2=0.25<0.33$, so condition $\mathbf{3}$ is met
ii) $\quad \mathrm{B}-\mathrm{L}=3.6 \mathrm{ft} \& 4 \mathrm{H}=2 \mathrm{ft}$, so condition $\mathbf{1}$ is met
$\mathrm{P}=1.2 \mathrm{ft} \& 2 \mathrm{H}=1 \mathrm{ft}$, so condition 2 is met $\mathrm{H} / \mathrm{L}=0.5 / 2.4=0.21<0.33$, so condition 3 is met
iii) $\mathrm{B}-\mathrm{L}=3 \mathrm{ft} \& 4 \mathrm{H}=3 \mathrm{ft}$, so condition 1 barely is met $\mathrm{P}=2.4 \mathrm{ft} \& 2 \mathrm{H}=2 \mathrm{ft}$, so condition 2 is met $\mathrm{H} / \mathrm{L}=0.75 / 3=0.25<0.33$, so condition 3 is met
iv) $\quad \mathrm{B}-\mathrm{L}=1.8 \mathrm{ft} \& 4 \mathrm{H}=4.8 \mathrm{ft}$, so condition $\mathbf{1}$ is not met $\mathrm{P}=2 \mathrm{ft} \& 2 \mathrm{H}=2.4 \mathrm{ft}$, so condition 2 is not met $\mathrm{H} / \mathrm{L}=1.2 / 4.2=0.28<0.33$, so condition 3 is met
v) $B-L=3 \mathrm{ft} \& 4 \mathrm{H}=8 \mathrm{ft}$, so condition 1 is not met $\mathrm{P}=2.4 \mathrm{ft} \& 2 \mathrm{H}=4 \mathrm{ft}$, so condition 2 is not met $\mathrm{H} / \mathrm{L}=2 / 3=0.67>0.33$, so condition $\mathbf{3}$ is not met

b) The calculations for part b) were done with an Excel spreadsheet. The results are summarized in the table below. Flow rates were calculated with both equations using the given values of $\mathrm{L}, \mathrm{P} \& \mathrm{H}$. In order to use equation (9), values were needed for $C_{1}, C_{2}, \& k_{b}$. All three are functions of $L / B$. Values for $\mathrm{C}_{1}, \& \mathrm{C}_{2}$ were obtained from Table 1, and values for $k_{b}$ were obtained from Figure 6.
$\mathrm{Q}_{9} \& \mathrm{Q}_{10}$, are the flow rates from equations (9) \& (10) respectively.

|  | $\underline{\text { i) }}$ | $\underline{\text { ii) }}$ | $\underline{\text { iii }}$ | $\underline{\text { iv }}$ | $\underline{\text { v) }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| eqn (10) ok? | yes | yes | yes | no | no |
| L/B | 0.5 | 0.4 | 0.5 | 0.7 | 0.5 |
| $\mathrm{C}_{1}$ | 0.0612 | 0.0317 | 0.0612 | 0.1602 | 0.0612 |
| $\mathrm{C}_{2}$ | 3.173 | 3.164 | 3.173 | 3.182 | 3.173 |
| $\mathrm{k}_{\mathrm{b}}$ | 0.010 | 0.009 | 0.010 | 0.013 | 0.010 |
| $\mathrm{Q}_{9}, \mathrm{cfs}$ | $\mathbf{2 . 3 3 0}$ | $\mathbf{2 . 7 6 6}$ | $\mathbf{6 . 3 3 9}$ | $\mathbf{1 8 . 3 8}$ | $\mathbf{2 7 . 5 3}$ |
| $\mathrm{Q}_{10}, \mathrm{cfs}$ | $\mathbf{2 . 2 3 7}$ | $\mathbf{2 . 7 0 8}$ | $\mathbf{6 . 1 6 3}$ | $\mathbf{1 7 . 3 3}$ | $\mathbf{2 4 . 4 9}$ |
| \% diff. | $4.0 \%$ | $2.1 \%$ | $2.8 \%$ | $5.7 \%$ | $11.1 \%$ |

Discussion of Results: The results from the two equations agree quite well for cases i), ii), \& iii), where the criteria for use of the simpler Equation (10) were met ( $\mathrm{B}-\mathrm{L} \geq 4 \mathrm{H}_{\max }, \mathrm{P} \geq 2 \mathrm{H}_{\max }, \& \mathrm{H} / \mathrm{L} \leq 0.33$ ). The results for the two equations differ considerably more for cases iv) and v ), where the criteria were not met.

Conclusions: For a contracted rectangular, sharp-crested weir, the simpler equation $\left[\mathbf{Q}=\mathbf{3 . 3 3}(\mathbf{L}-\mathbf{0 . 2} \mathbf{H})\left(\mathbf{H}^{3 / 2}\right)\right]$, appears to be adequate when the three criteria mentioned above are met. This equation is also acceptable to the U.S. Bureau of Land Reclamation for use when those criteria are met. The U. S. Bureau of Land Reclamation recommends the use of the slightly more complicated Kindsvater-Carter equation [equation (9)] for use when any of the three criteria given above for use of Equation (10) are not met.

## 7. Cipolletti Weirs

A cipolletti weir has a trapezoidal shape, as shown in figure 2(d) and in the figure below. The sides of the trapezoidal opening have a slope of 1 horizontal to 4 vertical as shown in the diagram.

cipolletti weir

The equation commonly used for discharge over a cipolletti weir, in U.S. units is as follows:
(U.S. units: Q in cfs, $\mathrm{L} \& \mathrm{H}$ in ft ): $\quad \mathbf{Q}=\mathbf{3 . 3 6 7} \mathbf{L} \mathbf{H}^{3 / 2}$

To be used only for: $\mathbf{0 . 2} \mathbf{f t} \leq \mathbf{H} \leq \mathbf{2 f t}$ and $\mathbf{H} \leq \mathbf{L} / \mathbf{3}$

Equation (12) can be converted to S.I. units to give the following:
(S.I. units: Q in $\mathrm{m}^{3} / \mathrm{s}$, L \& H in m ): $\quad \mathbf{Q}=\mathbf{1 . 8 4} \mathbf{L ~ H}^{3 / 2}$

The conditions given for equation (12) apply to equation (13) also
Example \#6: (a) Calculate the minimum and maximum flow rates for $0.2 \mathrm{ft} \leq$ $\mathrm{H} \leq 0.7 \mathrm{ft}$ for a cipolletti weir, and for a contracted rectangular weir each with a weir length of $\mathrm{L}=2 \mathrm{ft}$. Assume that the channel width, B is 7 ft and $\mathrm{P}=2 \mathrm{ft}$, so that both weirs are fully contracted.
(b) Compare with the flow rates for $\mathrm{H}=0.2 \mathrm{ft}$ and $\mathrm{H}=0.7 \mathrm{ft}$ for a suppressed rectangular weir in a 2 ft wide channel.

Solution: Equation (12) will be used for the cipolletti weir calculations. Equation (10) will be used for the contracted rectangular weir calculations, since it is fully contracted. Equation (8) will be used for the suppressed weir calculations since the conditions for its use are met. All that needs to be done is
to substitute $\mathrm{L}=2 \mathrm{ft}$ and the two different values for H into the three equations. This was done with Excel. The results are summarized in the table below.

|  | Cipolletti <br> Q, cfs | Contracted <br> Rectangular <br> Q, cfs | Suppressed <br> Rectangular <br> Q, cfs |
| :---: | :---: | :---: | :---: |
| 0.2 | 0.602 | 0.584 | 0.596 |
| 0.7 | 3.944 | 3.627 | 3.901 |

Discussion of Results: For this range, the flow rates over the cipolletti and suppressed rectangular weir are nearly the same for the same weir length and head. The flow rate over the contracted rectangular weir is slightly less for the same weir length and head.

## 8. Installation \& Use Guidelines for Sharp-Crested Weirs

A summary of installation and measurement guidelines for sharp-crested weirs extracted from Water Measurement Manual, is given in this section.
a) The upstream face of the weir plates and bulkhead should be plumb, smooth, and normal to the axis of the channel.
b) The entire crest should be level for rectangular and trapezoidal shaped weir openings, and the bisector of V -notch angles should be plumb.
c) The top thickness of the crest and side plates should be between 0.03 and 0.08 inch.
d) The upstream edges of the weir opening plates must be straight and sharp. Edges of plates require machining or filing perpendicular to the upstream face to remove burrs or scratches and should not be smoothed off with abrasive cloth or paper. Avoid knife edges because they are a safety hazard and damage easily
e) The overflow sheet or nappe should touch only the upstream faces of the crest and side plates.
f) Maximum downstream water surface level should be at least 0.2 ft below crest elevation. However, when measuring close to the crest, frequent observations are necessary to verify that the nappe is continually ventilated without waves periodically filling the under nappe cavity.
g) The measurement of head on the weir is the difference in elevation between the crest and the water surface at a point located upstream from the weir a distance of at least four times the maximum head on the crest.
h) The approach to the weir crest must be kept free of sediment deposits.
i) If the weir crest length is greater than $50 \%$ of the approach channel width, then ten average approach flow widths of straight, unobstructed approach are required.
j) If the weir crest length is less than $50 \%$ of the approach channel width, then twenty average approach flow widths of straight, unobstructed approach are required.
k) If upstream flow is below critical depth, a jump should be forced to occur. In this case, thirty measuring heads of straight, unobstructed approach after the jump should be provided.

1) The minimum head over the weir should be 0.2 ft .

## For V-notch Weirs:

All of the general conditions, a) through 1) above, apply. Also:
a) For a fully contracted V-notch weir, the maximum measured head, H , should be less than 1.25 ft .
b) For a fully contracted V-notch weir, H/B should be less than 0.2 .
c) The average width of the approach channel, B, should always be greater than 3 ft for a fully contracted V-notch.
d) The V-notch of the weir should always be at least 1.5 ft above the invert of the weir pool for a fully contracted V-notch.

## For Rectangular, Sharp-crested Weirs:

All of the general conditions, a) through 1) above, apply. Also:
a) The crest length, L, should be at least 6 inches.
b) The crest height, P , should be at least 4 inches.
c) Values of $\mathrm{H} / \mathrm{P}$ should be less than 2.4 .

## For Cipollette Weirs:

All of the general conditions, a) through 1) above, apply. Also:
a) Values of $\mathrm{H} / \mathrm{L}$ should be less than 0.333
b) Values of H should be less than 0.5 P .
c) $\mathrm{B}-\mathrm{L} \geq 4 \mathrm{H}$

## 9. Summary

Sharp-crested weirs are commonly used for flow rate measurement in open channels. Four types of sharp-crested weirs: V-notch, suppressed rectangular, contracted rectangular, and cipolletti, are covered in detail in this course.
Emphasis is on calculation of flow rate over a weir for given head over the weir and weir/channel dimensions. For each of the four types of sharp-crested weir, a general equation with a wide range of applicability is presented and discussed along with equations and/or graphs as needed for use with the main equation. Also, for each of the four types of sharp-crested weir, a simpler equation is presented along with a set of conditions under which the simpler equation can be used. Several worked examples are included covering all four types of weirs. Practical installation and use guidelines for sharp-crested weirs are presented.

## 10. References and Websites

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2. LMNO Engineering, Research and Software, Ltd website. Contains equations and graphs for open channel flow measurement. http://www.lmnoeng.com/Weirs
